Schutz 8.1. ϕ spherically symmetric $\to$ $\phi = \phi(r)$

$\vec{\nabla}\phi(r)$ is a vector / 1-form.

Gauss's law tells us

$$\int_{\text{Sphere}} \vec{\nabla}\phi(r) \cdot d\vec{a} = \int_{\text{Sphere.}} \vec{\nabla}^2 \phi(r)\, d^3x$$

$$\vec{\nabla}\phi(r) \cdot d\vec{a} = \frac{d\phi}{dr},$$

The prompt gives $\vec{\nabla}^2\phi = 4\pi G\rho$, so we have

$$\frac{d\phi}{dr}(\cancel{4\pi} r^2) = \cancel{4\pi} G \int \rho(\vec{x})\, d^3x$$

$$\Rightarrow \boxed{\frac{d\phi(r)}{dr} = \frac{G}{r^2}\int \rho(\vec{x})\, d^3x}$$

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